EDUSAT PROGRAMME 15

Turbomachines (Unit 3)

Axial flow compressors and pumps

Axial flow compressors and pumps are power absorbing turbomachines. These machines absorb external power and thereby increase the enthalpy of the flowing fluid. Axial flow turbomachines use large quantity of fluid compared to mixed and centrifugal type of turbomachines. However, the pressure rise per stage is lower in case of axial flow turbomachines than mixed and centrifugal flow turbomachines. Axial flow compressors are used in aircraft engines, stand alone power generation units, marine engines etc.

Design of axial flow compressors is more critical than the design of turbines (power generating turbomachines). The reason is that in compressors the flow moves in the direction of increasing pressure (adverse pressure gradient). If the flow is pressurized by supplying more power, the boundary layers attached to the blades and casing get detached and reverse flow starts which leads to flow instability leading to possible failure of the machine. However, in turbines the fluid moves in the decreasing pressure (favourable pressure gradient). To transfer a given amount of energy more number of stages are required in compressors than in turbines. Generally, the fluid turning angles are limited to 20˚ in compressors and is 150˚ to 165˚ in case of turbines. Thus the pressure rise per stage is limited in case of compressors. Fluid flows in direction of increasing pressure, this increases the density of the fluid thus the height of the blade decreases from the entrance to the exit. Axial flow compressors have inlet guide vanes at the entrance and diffuser at the exit.

Figure 1 shows the flow through the compressor and fig. 2 shows the corresponding inlet and exit velocity triangles. Generally for a compressor the angles are defined with respect to axial direction (known as air angles). It can be seen that the fluid turning angle is low in case of compressors. Figure 3 shows the flow through the turbine blade and fig. 4 the corresponding inlet and exit velocity triangles for a turbine.

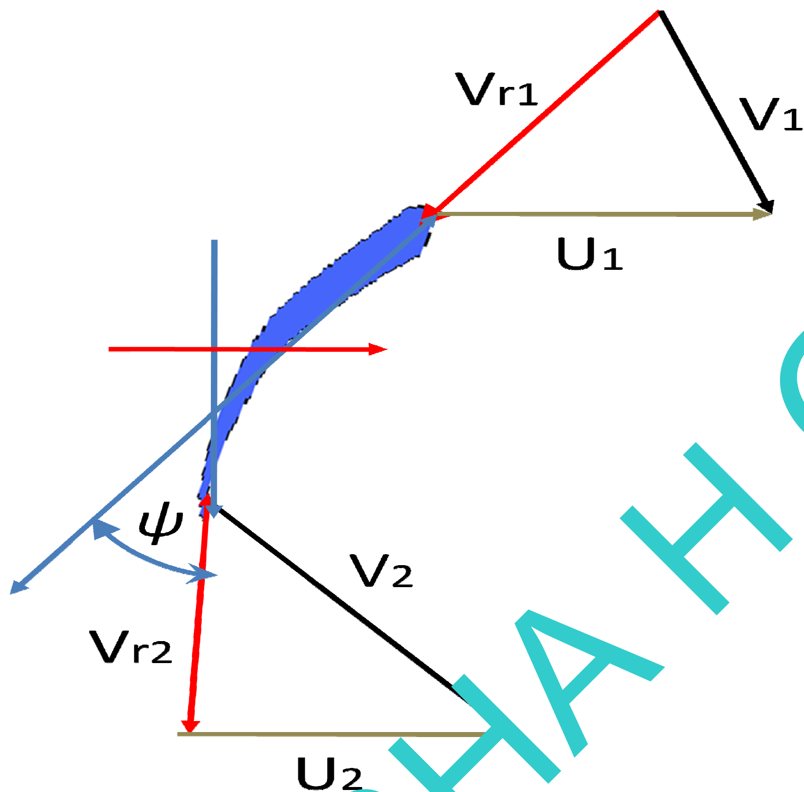


Fig. 1 Flow across a compressor blade

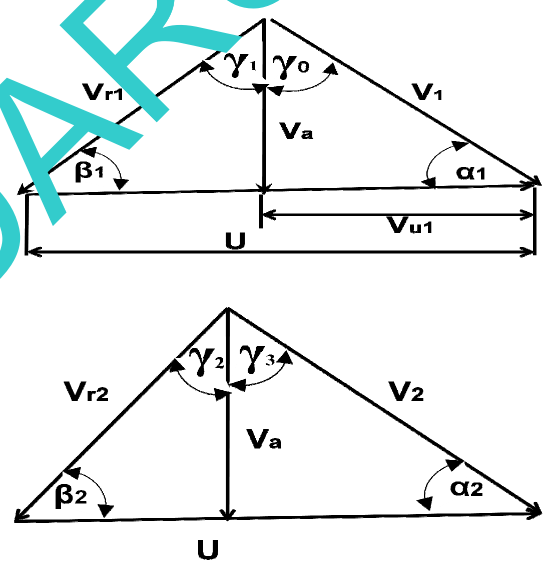


Fig. 2 Inlet and exit velocity triangles

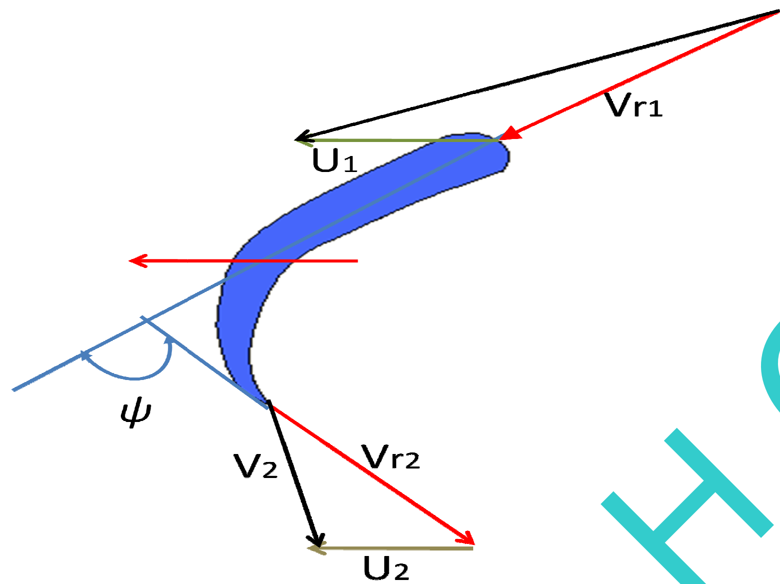


Fig. 3 Flow across a turbine blade

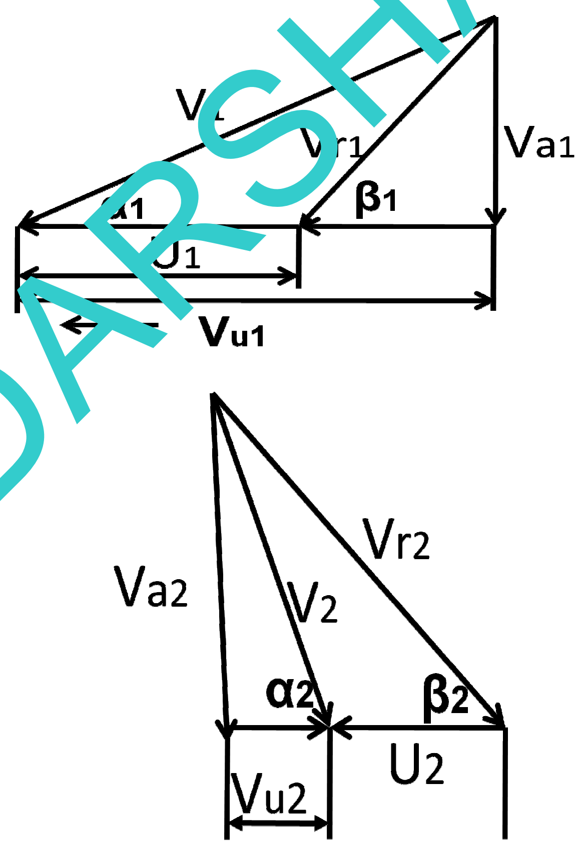


Fig. 4 Inlet and exit velocity triangles for turbine

Expression for degree of reaction for axial flow compressor

Degree of reaction is defined as a measure of static enthalpy rise that occurs in the rotor expressed as a percentage of the total static enthalpy across the rotor.

$$R = \frac{Static\ \ enthalpy\ \ rise\ \ in\ \ rotor}{Static\ \ enthalpy\ \ rise\ \ in\ \ stage}$$

Assumptions:

1. The analysis is for 2-dimensional flow
2. The flow is assumed to take place at a mean blade height where blade peripheral velocities at inlet and outlet are same and there is no flow in the radial direction
3. It is common to express blade angles w.r.t. axial direction in case of axial flow compressor (air angles)
4. Axial velocity is assumed to remain constant

Figure 2 shows the inlet and exit velocity triangles for axial flow compressors.

The work done on the compressor = Change in stagnation enthalpy

$$\Delta h_0 = U(V_{u2} - V_{u1}) --- 1$$

For an axial flow TM, From inlet velocity triangle

$$U = V_a tan\gamma_1 + V_a tan\gamma_0 = V_a(tan\gamma_1 + tan\gamma_0) -- 2$$

Similarly from the exit velocity triangle

$$U = V_a(tan\gamma_2 + tan\gamma_3) -- 3$$

Equating eq. 2 and eq. 3 and simplifying

$$tan\gamma_1 - tan\gamma_2 = tan\gamma_3 - tan\gamma_0 --- 4$$

$$R = \frac{V_{r1}^2 - V_{r2}^2}{2(h_{02} - h_{01})} --- 5$$

$$V_{r1}^{\ 2} = V_a^{\ 2} + V_a^{\ 2} tan^2\gamma_1$$

$$V_{r2}^{\ 2} = V_a^{\ 2} + V_a^{\ 2} tan^2\gamma_2$$

$$V_{r1}^{\ 2} - V_{r2}^{\ 2} = V_a^{\ 2}(tan^2\gamma_1 - tan^2\gamma_2) --- 6$$

$$h_{02} - h_{01} = U(V_{u2} - V_{U1})$$

$$V_{u2} = V_a tan\gamma_3 \quad and \quad V_{u1} = V_a tan\gamma_0$$

$$\Delta h_0 = UV_a(tan\gamma_3 - tan\gamma_0) --- 7$$

Substituting eq. 6 and 7 in eq.5

$$R = \frac{V_a^{\ 2}(tan^2\gamma_1 - tan^2\gamma_2)}{2UV_a(tan\gamma_3 - tan\gamma_0)}$$

$$tan\gamma_1 - tan\gamma_2 = tan\gamma_3 - tan\gamma_0 --- 4$$

$$R = \frac{V_a^{\ 2}(tan^2\gamma_1 - tan^2\gamma_2)}{2UV_a(tan\gamma_1 - tan\gamma_2)}$$

$$R = \frac{V_a(tan\gamma_1 + tan\gamma_2)}{2U} ---- 8$$

$$R = \frac{V_a}{2U}(cot\beta_1 + cot\beta_2) --- 9$$

$$R = \frac{V_a}{2U}\left(\frac{1}{tan\beta_1} + \frac{1}{tan\beta_2}\right)$$

$$R = \frac{V_a}{2U}\left(\frac{tan\beta_1 + tan\beta_2}{tan\beta_1 tan\beta_2}\right) --- 10$$

- **Velocity triangles for different values of degree of reaction**

Velocity triangles are drawn for axial flow turbomachines in which $U_1=U_2=U$

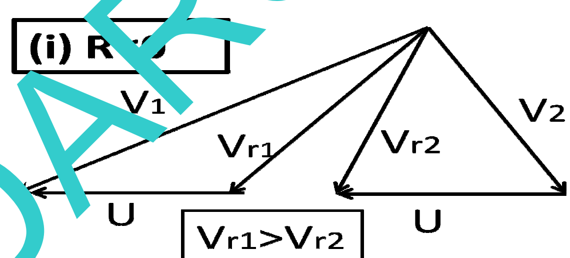
$$R=\frac{V_{r2}^{2}-V_{r1}^{2}}{\left(V_{1}^{2}-V_{2}^{2}\right)+\left(V_{r2}^{2}-V_{r1}^{2}\right)}$$

$$E=WD=\left(\frac{\left(V_{1}^{2}-V_{2}^{2}\right)+\left(V_{r2}^{2}-V_{r1}^{2}\right)}{2}\right)$$

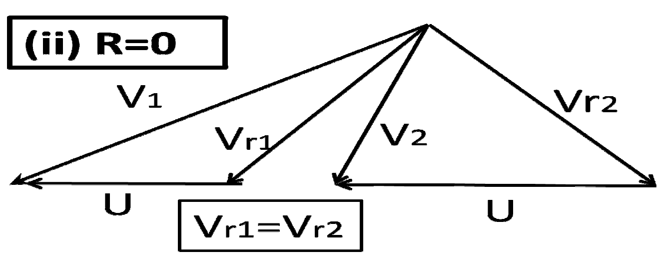
(i) When $R < 0$ (R is negative)

R becomes – ve when $V_{r1}>V_{r2}$

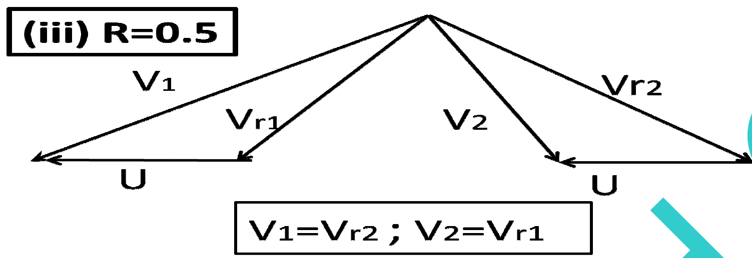
E or WD can be positive



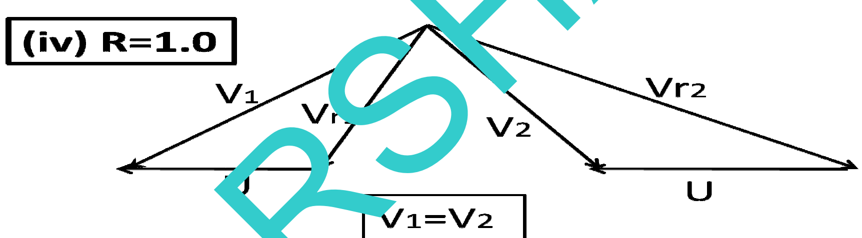
(ii) When $R=0$; $V_{r1}=V_{r2}$; Impulse TM; No static pressure change across rotor



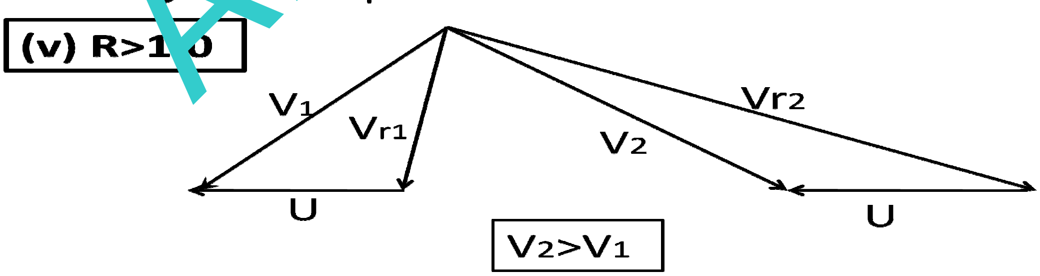
(iii) When R=0.5; V1=Vr2 and V2=Vr1; Impulse TM; 50% by impulse and 50% by reaction, Symmetrical velocity triangle; α1=β2; α2=β1



(iv) When R=1.0; V1=V2; purely reaction TM; Energy transformation occurs purely due to change in relative kinetic energy of fluid



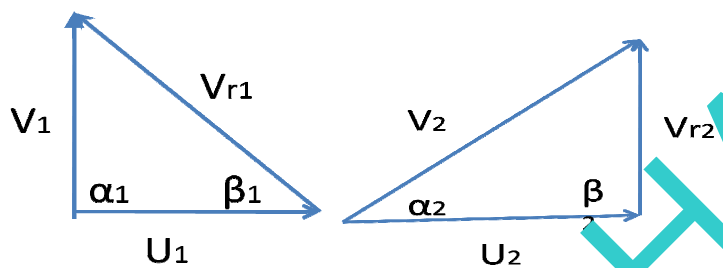
(v) When R>1.0; V2>V1; Energy transformation can be negative or positive



Problem 1

In a mixed flow pump absolute fluid velocity at the inlet is axial and equal to radial velocity at the exit. Inlet hub diameter is 80 mm and impeller tip diameter is 250 mm. Pump speed is 3000 rpm. Find the degree of reaction and the energy input to the fluid, if the relative velocity at the exit equals the inlet tangential blade speed.The fluid leaves the rotor in the radial direction Given: V1 = Va1= Vrd2 ; D1= 80 mm, D2= 250 mm, N=3000 rpm, R = ?, WD=?, Vr2=U1

The next step is to draw the velocity triangles



$$V_{r2} = U_1 = 12.57 m/s; \quad V_2 = \sqrt{U_2^2 + V_{r2}^2} = 41.23 m/s$$

$$V_{rd2} = U_1 = V_1 = 12.57 m/s; \quad V_{r1} = \sqrt{V_1^2 + U_1^2} = 17.78 m/s$$

$$E = WD = -U_2 V_{u2} = 39.2 \times 39.2 = \underline{1.543 KJ/kg}$$

$$R = \frac{\left(U_1^2 - U_2^2\right) - (V_{r1}^2 - V_{r2}^2)}{2 \times E}$$

$$R = \underline{0.5}$$

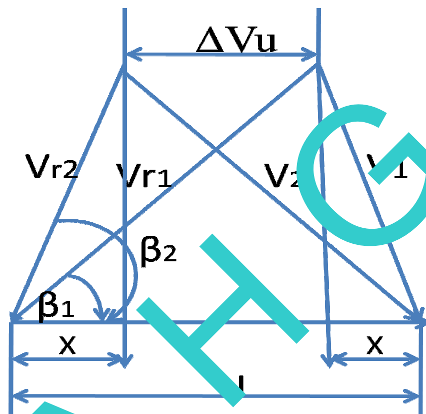
Problem 2

The total power input at a stage in an axial flow compressor with symmetric inlet and outlet velocity triangles (R=0.5) is 27.85 kJ/kg of air flow. If the blade speed is 180m/s throughout the rotor, draw the velocity triangles and compute the rotor blade angles. Do you recommend the use of such compressor?. Assume the axial velocity component to be 120m/s.

Given: Symmetric velocity triangles with R = 0.5; P= 27.85 kJ/kg of air flow; U=180m/s; Va=120m/s; β1=? and β2=?; Do you recommend such compressor? Draw the velocity triangles.

Since the R=0.5 and velocity triangles are symmetrical

α1=β2; α2=β1; V1=Vr2 and V2=Vr1



$$\boldsymbol{WD = U(\Delta\ V_u) = 27850 kJ/Kg}$$
$$\boldsymbol{\Delta V_u = 154.72 m/s}$$

$$\boldsymbol{U - 2 \times X = \Delta V_u; \quad X = (U - \Delta V_u)/2}$$
$$\boldsymbol{X = 12.64}$$

$$\boldsymbol{\beta_2 = tan^{-1}\left(\frac{120}{12.64}\right) = \underline{83.98^\circ}}$$

$$\boldsymbol{\beta_1 = tan^{-1}\left(\frac{120}{180 - 12.64}\right) = \underline{35.64^\circ}}$$

$$\boldsymbol{\beta_2 - \beta_1 = 48.34^\circ}$$

Problem 3

The axial component of air velocity at the exit of the nozzle of an axial flow reaction stage is 180m/s, the nozzle inclination to the direction of rotation is 27˚. Find the rotor blade angles if the degree of reaction is 50% and the blade speed is 180m/s. Also, for the same blade speed, axial velocity and nozzle angle find degree of reaction, if the absolute velocity at the rotor outlet should be axial and equal to axial velocity at the inlet.

Given: 1) Va1=180m/s, α1=27˚, R = 50%, U=180m/s, β1=?, β2=?

2) U=180m/s, Va1=180m/s, α1=27˚, R=?, if V2=Va2=Va1

Sol: 1) R = 50%, V1=Vr2, V2=Vr2, α1=β2 and α2=β1

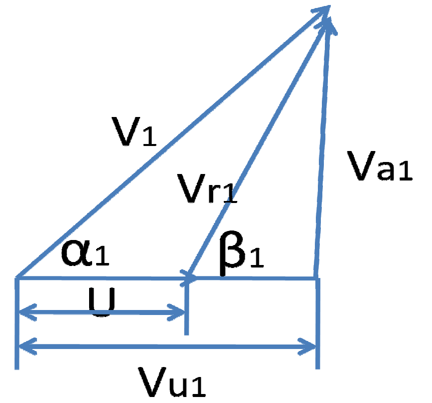
Inlet velocity triangle is shown in the figure

$$V_1 sin\alpha_1 = 180; \quad V_1 = 396.48 m/s$$

$$V_1 cos\alpha_1 - U = 173.26 m/s$$

$$\beta_1 = tan^{-1}\frac{180}{173.26} = 46.09^{\circ}$$

$$\alpha_1 = \beta_2 \quad \beta_2 = 27^{\circ}$$



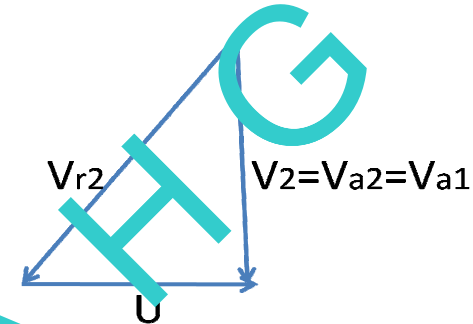
From the exit velocity triangles

$$\beta_2 = 45^{\circ}, \; V_{r2} = 254.55 m/s$$

$$V_{r1} = \sqrt{173.26^2 + 180^2} = 249.83 m/s$$

$$V_1 = 396.48 m/s \quad V_2 = 180 m/s$$

$$R = \frac{V_{r2}^{\;2} - V_{r1}^{\;2}}{\left(V^2{}_1 - V_2^{\;2}\right) + \left(V_{r2}^{\;2} - V_{r1}^{\;2}\right)} = 0.0198$$



Turbines- Utilization factor

1.Utilization factor is defined only for PGTM- Turbines

2.Adiabatic efficiency is the quantity of interest in turbines

3.Overall efficiency is product of adiabatic efficiency and mechanical efficiency

4.Mechanical efficiency of majority of TM's is nearly 100%

5.Therefore, overall efficiency is almost equal to adiabatic efficiency

6. However, adiabatic efficiency is product of utilization factor (diagram efficiency) and efficiency associated with various losses.

7.Utilization factor deals with what is maximum energy that can be obtained from a turbine without considering the losses in the turbine

8.Utilization factor is the ratio of ideal work output to the energy available for conversion to work

9.Under ideal conditions it should be possible to utilize all the K.E. at inlet and increase the K.E. due to reaction effect

10. The ideal energy available for conversion into work

$$W_a = \frac{\left[V_1^2 + \left(U_1^2 - U_2^2\right) - \left(V_{r1}^2 - V_{r2}^2\right)\right]}{2}$$

11. The work output given by Euler's Turbine Equation is

$$W = \frac{\left[\left(V_1^2 - V_2^2\right) + \left(U_1^2 - U_2^2\right) - \left(V_{r1}^2 - V_{r2}^2\right)\right]}{2}$$

11. Utilization factor is given by

$$\varepsilon = \frac{W}{W_a} = \frac{\left[\left(V_1^2 - V_2^2\right) + \left(U_1^2 - U_2^2\right) - \left(V_{r1}^2 - V_{r2}^2\right)\right]}{\left[V_1^2 + \left(U_1^2 - U_2^2\right) - \left(V_{r1}^2 - V_{r2}^2\right)\right]}$$

12. Utilization factor for modern TM's is between 90% to 95%

Relation between utilization factor and degree of reaction

Utilization factor is given by

$$\varepsilon = \frac{W}{W_a} = \frac{\left[\left(V_1^2 - V_2^2\right) + \left(U_1^2 - U_2^2\right) - \left(V_{r1}^2 - V_{r2}^2\right)\right]}{\left[V_1^2 + \left(U_1^2 - U_2^2\right) - \left(V_{r1}^2 - V_{r2}^2\right)\right]}$$

The degree of reaction is given by

$$R = \frac{\left(U_1^2 - U_2^2\right) - \left(V_{r1}^2 - V_{r2}^2\right)}{\left(V_1^2 - V_2^2\right) + \left(U_1^2 - U_2^2\right) - \left(V_{r1}^2 - V_{r2}^2\right)} = \frac{\left(U_1^2 - U_2^2\right) - \left(V_{r1}^2 - V_{r2}^2\right)}{2 \times E}$$

$$X = \left(U_1^2 - U_2^2\right) - \left(V_{r1}^2 - V_{r2}^2\right)$$

$$R = \frac{X}{\left(V_1^2 - V_2^2\right) + X}$$

$$X = \frac{R\left(V_1^2 - V_2^2\right)}{1 - R}$$

Substituting the value of X in the expression for utilization factor and simplifying

$$\varepsilon = \frac{\left(V_1^2 - V_2^2\right)}{\left(V_1^2 - RV_2^2\right)}$$

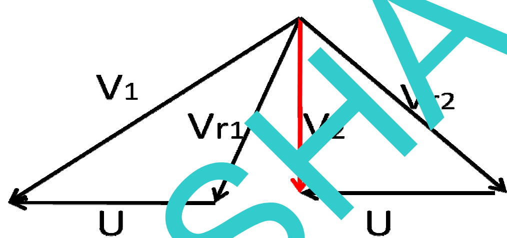
The above equation is valid for single rotor under the conditions where Euler's turbine equations are valid. The above equation is invalid when $R = 1$. The above equation is valid in the following range of R $0 \le R < 1$

Maximum Utilization factor

Utilization factor is given by

$$\varepsilon = \frac{\left(V_1^2 - V_2^2\right)}{\left(V_1^2 - RV_2^2\right)}$$

Utilization factor maximum if the exit absolute velocity is minimum. This is possible when the exit absolute velocity is in axial direction.



$$V_2 = V_1 \sin\alpha_1$$

$$\varepsilon_m = \frac{V_1^2 - V_1^2 \sin^2\alpha_1}{V_1^2 - RV_1^2 \sin^2\alpha_1}$$

$$\varepsilon_m = \frac{\cos^2\alpha_1}{1 - R\sin^2\alpha_1}$$

Maximum utilization factor is given by

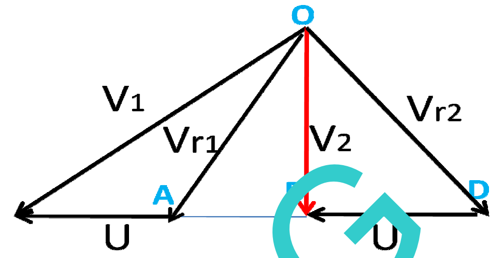
$$\varepsilon_m = \frac{\cos^2\alpha_1}{1 - R\sin^2\alpha_1} \qquad \varepsilon_m = 1 \text{ when } \alpha_1 = 0$$

Maximum Utilization factor for impulse turbine

For an impulse turbine R = 0, thus $\varepsilon_m = \cos^2\alpha_1$
From the velocity triangles OAB and OBD are similar. Thus AB = U,

$$V_1\cos\alpha_1 = U + U = 2U$$

$$\frac{U}{V_1} = \frac{\cos\alpha_1}{2} = \text{Speed Ratio} = \varphi$$



α_1 is made as small as possible(15°-20°)

Maximum Utilization factor for 50% reaction turbines

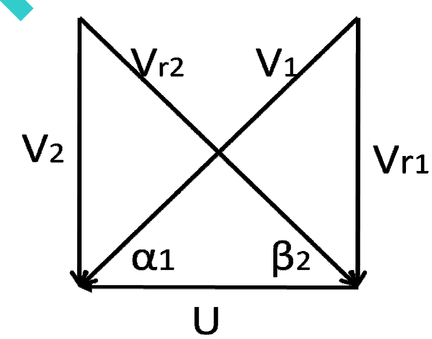
50% reaction turbines have V1=Vr2, V2=Vr1, α1=β2 and α2=β1 and for maximum utilization V2 must be in axial direction. The corresponding velocity triangles are

$$\varepsilon_m = \frac{\cos^2\alpha_1}{1 - R\sin^2\alpha_1}$$

$$R = 0.5, \quad \varepsilon_m = \frac{\cos^2\alpha_1}{1 - 0.5\sin^2\alpha_1}$$

$$\text{Also, } V_1\cos\alpha_1 = U$$

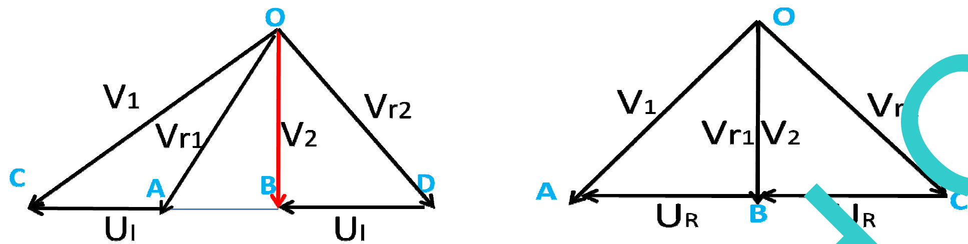
$$\frac{U}{V_1} = \cos\alpha_1 = \varphi = \text{speed ratio}$$



Comparison between Impulse and 50% reaction turbine at maximum utilization

A) When both have same blade speed

Let UI and UR be the blade speeds of impulse and 50% reaction turbines. The velocity triangles for maximum utilization are



$$E_I = U_I V_{u1}$$

From velocity triangle $V_{u1} = 2U_I$

$$E_I = 2U_I^2 \;---1$$

For 50% reaction turbine

$$E_R = U_R V_{u1}$$

But $V_{u1} = U_R$

$$E_R = U_R^2 \;---2$$

Comparing eq. 1 and 2, it is clear that impulse turbine transfers twice the amount of energy per unit mass than 50% reaction turbine for the same blade speed when utilization is maximum.

- However, 50% reaction turbines are more efficient than impulse turbines.
- But 50% reaction turbines transfer half the energy compared to impulse turbines.
- If only 50% reaction turbines are used more stages are required or if only impulse turbines are used stages are less but efficiency is low.
- In steam turbines where large pressure ratio is available it is common to use one or two impulse stages followed by reaction stages.

Comparison between Impulse and 50% reaction turbine at maximum utilization

b) When both have same energy transfer

$$E_R = E_I$$
$$U_R^2 = 2U_I^2$$
$$U_R = \sqrt{2U_I^2} = 1.414U_I - - - 3$$

c) When V1 and α1 are same in both TM's

Speed ratio for impulse and 50% reaction stage for maximum utilization

$$\frac{U_I}{V_1} = \varphi = \frac{\cos\alpha_1}{2}; \quad 2U_I = \cos\alpha_1 - -4$$
$$\frac{U_R}{V_1} = \varphi = \cos\alpha_1; \quad U_R = V_1\cos\alpha_1 - -5$$
$$U_R = 2U_I - -6$$

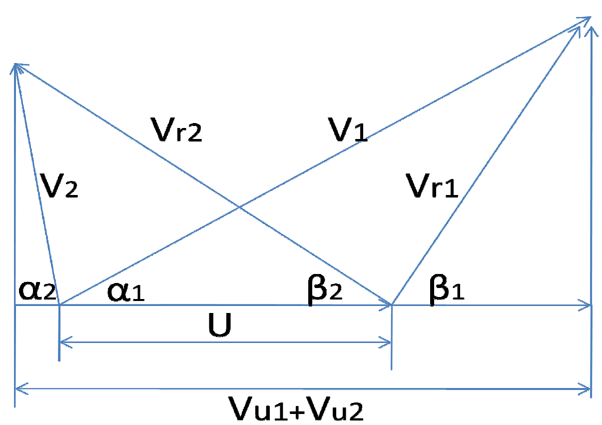
Problem 1

Find an expression for the utilization factor for an axial flow impulse turbine stage which has equiangular rotor blades, in terms fixed inlet blade angle and speed ratio ϕ.

Given: Equiangular rotor blades β1=β2, axial flow turbine U1=U2=U, Impulse turbine R = 0, Vr1=Vr2, find expression for utilization factor in terms of α1 and ϕ.

From the velocity triangles

$$V_2^2 = V_{r2}^2 + U^2 - 2UV_{r2}\cos\beta_2$$
$$V_1^2 = V_{r1}^2 + U^2 - 2UV_{r2}\cos(180 - \beta_2)$$



$$V_1^2 - V_2^2 = 4UV_{r1}\cos\beta_1 = 4U(V_1\cos\alpha_1 - U)$$

$$\varepsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

$$\varepsilon = \frac{V_1^2 - V_2^2}{V_1^2} = \frac{4U(V_1\cos\alpha_1 - U)}{V_1^2}$$

$$\underline{\varepsilon = 4\varphi\ (\cos\alpha_1 - \varphi)}$$

$$where,\quad \varphi = \frac{U}{V_1}$$

Problem 2

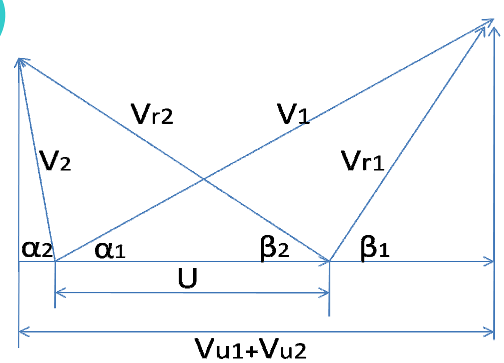
For a 50% degree of reaction axial flow turbomachine, the inlet fluid velocity is 230m/s; outlet angle of inlet guide blade 30°; inlet rotor angle 60° and the outlet rotor angle is 25°. Find the utilization factor, axial thrust and the power output/unit mass flow.

Given: R=50%; V1=230m/s; α1=30°; β1=60°; β2=25°

ε = ?; Fax=?; P = ?

Although R = 50%, α1≠ β2; as Va1≠Va2

Inlet and exit velocity triangles are drawn



$$V_{a1} = V_1\sin\alpha_1 = 115m/s$$

$$V_{r1} = \frac{V_{a1}}{\sin\beta_1} = 132.8m/s$$

$$U = V_1\cos\alpha_1 - V_{r1}\cos\beta_1 = 132.8m/s$$

$$R = \frac{V_{r2}^2 - V_{r1}^2}{(V_1^2 - V_2^2) + (V_{r2}^2 - V_{r1}^2)} = 0.5$$

$$V_{r1}^2 + V_1^2 = V_{r2}^2 + V_2^2 = 132.8^2 + 230^2 = 70535.8$$

$$V_2^2 = V_{r2}^2 + U^2 - 2UV_{r2}\cos\beta_2$$

$$2V_{r2}^2 - 240.72V_{r2} + 17635.8 = 70535.8$$

$$V_{r2}^2 - 120.36V_{r2} - 26450 = 0$$

$$V_{r2} = 233.6m/s$$

$$V_{r2}^{\ 2} + V_2^{\ 2} = 70535.8$$

$$V_2 = 126.36 m/s$$

$$\varepsilon = \frac{V_1^{\ 2} - V_2^{\ 2}}{V_1^{\ 2} - RV_2^{\ 2}} = \frac{230^2 - 126.36^2}{230^2 - 0.5 \times 126.36^2} = 0.822$$

$$Axial \quad Thrust = \dot{m}\left(V_{a1} - V_{a2}\right)$$

$$\frac{F_{ax}}{\dot{m}} = \left(V_{a1} - V_{a2}\right) = V_1 Sin\alpha_1 - V_{r2} sin\beta_2$$

$$\frac{F_{ax}}{\dot{m}} = 16.28 \frac{N}{\frac{kg}{s}}$$

$$P = \dot{m}U\left(V_{u1} - V_{u2}\right)$$

$$V_{u1} = V_1 cos\alpha_1 = 199.98 m/s$$

$$V_{u2} = V_{r2} cos\beta_2 - U = 78.91 m/s$$

$$\frac{P}{\dot{m}} = 132.8\left(199.98 + 78.91\right) = 37 KJ/kg/s$$

UNIT 4

Thermodynamics of fluid

Sonic velocity and Mach number

1.Sonic velocity is the speed of propagation of pressure wave in a medium.
2.The speed of sound in a fluid at a local temperature for isentropic flow is given by where γ is the ratio of specific heats = 1.4, R is characteristics gas constant = 287 J/kg K and T is the local temperature in kelvins. At 15° the speed of sound is 340 m/s.

$$c = \sqrt{\gamma R T}$$

3.As altitude increases temperature decreases and speed of sound decreases.
4.Mach Number is defined as the ratio of local velocity of fluid to the sonic velocity of sound in that fluid

$$M = \frac{V}{c} = \frac{V}{\sqrt{\gamma R T}}$$

5. Many turbines and compressors experience high Mach numbers
6.High Mach numbers give rise to some special problems such as shock waves which leads to irreversibility and cause loss in stagnation pressure and increase in entropy
7.Using continuity equation, Euler's equation and isentropic equation following two equations are derived

$$\frac{dV}{V} = -\frac{dp}{p\,\gamma\,M^2} - - - 1$$

$$\frac{dA}{A} = \frac{dp}{p}\left(\frac{1 - M^2}{\gamma M^2}\right) - - - 2$$

8.The above equations decide the variations in velocity, pressure and area for different Mach numbers
9.The three basic classifications are a) Subsonic flow M<1
b) Sonic flow M=1 c) Supersonic flow M>1

Classification fluid flow based on Mach number

a) Subsonic flow (M<1) :

Nozzle

$$when \quad \frac{dp}{p} \quad is\ negative \quad \frac{dA}{A} \quad is\ negative$$

$$area \quad decreases\ and\ velocity\ increases$$

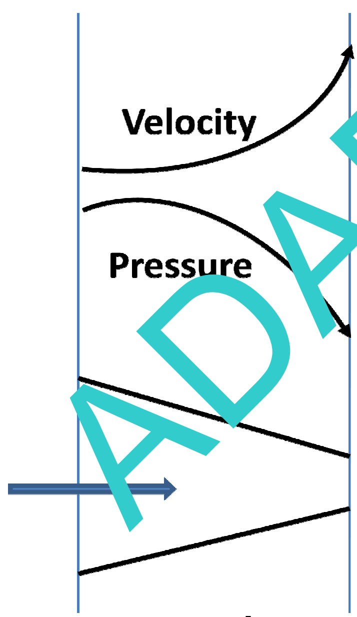
$$\frac{dV}{V} = -\frac{dp}{p\,\gamma\,M^2} ---1$$

$$\frac{dA}{A} = \frac{dp}{p}\left(\frac{1-M^2}{\gamma M^2}\right) ---2$$

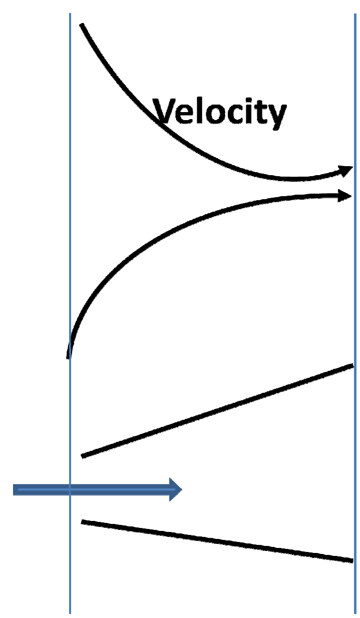
Diffuser

$$when \quad \frac{dp}{p} \quad is\ increases \quad \frac{dA}{A} \quad is\ positive$$

$$area \quad increases\ and\ velocity\ decreases$$



Nozzle



Diffuser

b) Supersonic flow (M>1) :

Divergent nozzle

when $\frac{dp}{p}$ *is negative* $\frac{dA}{A}$ *is positive*

area increases and velocity increases

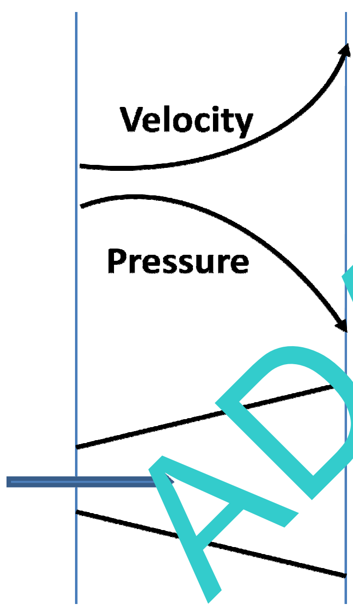
$$\frac{dV}{V} = -\frac{dp}{p\,\gamma\,M^2} - - - 1$$

$$\frac{dA}{A} = \frac{dp}{p}\left(\frac{1-M^2}{\gamma M^2}\right) - - - 2$$

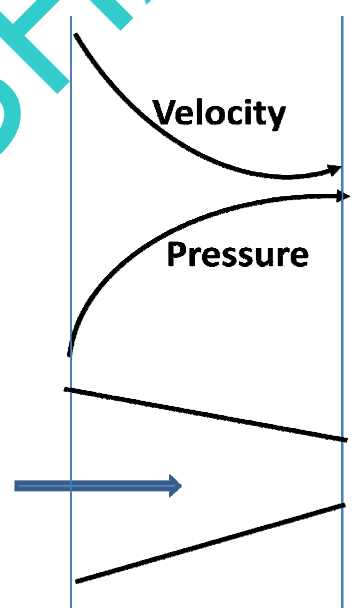
Convergent Diffuser

when $\frac{dp}{p}$ *is positive* $\frac{dA}{A}$ *is negative*

area decrease and velocity decreases



Divergent Nozzle



Convergent Diffuser

C) Sonic flow (M=1) :

$$\frac{dV}{V} = -\frac{dp}{p\,\gamma\, M^2} ---1$$

$$\frac{dA}{A} = \frac{dp}{p}\left(\frac{1-M^2}{\gamma M^2}\right) ---2$$

$\frac{dA}{A}$ ***is zero***

area is constant and velocity is sonic

At the throat portion of the convergent divergent nozzle the velocity is sonic.

Static and Stagnation states

Turbomachines involve the use of compressible and incompressible fluids. In compressible TM's fluid move with velocities more than Mach one at many locations. In incompressible TM's the fluid velocities are generally low, however, K.E. and P.E. of the moving fluid are very large and cannot be neglected To formulate equations based on actual state of the fluid based on laws of thermodynamics two states are used.

The states are static state and stagnation states.

Static State

If the measuring instrument is static with respect to the fluid, the measured quantity is known as static property. The measured static property could be pressure, velocity temperature, enthalpy etc.,. The state of the particle fixed by a set of static properties is called static state.

Stagnation State

It is defined as the terminal state of a fictitious, isentropic work-free and steady flow process during which the final macroscopic P.E. and K.E. of the fluid particle are reduced to zero.

Real process does not lead to stagnation state because no real process is isentropic. Stagnation property changes provide ideal value against which the real machine performance can be compared. It is possible to obtain stagnation properties in terms of static properties by using the definition of stagnation state. Consider the steady flow process given by the first law of thermodynamics.

$$q + \left(h_i + \frac{V_i^2}{2} + gZ_i \right) = w + \left(h_o + \frac{V_o^2}{2} + gZ_o \right)$$

$$q - w = \Delta h + \Delta ke + \Delta pe$$

Static state is the initial state in a fictitious isentropic work free , steady flow process and the stagnation state is the terminal state in which the ke and pe are reduced to zero, one can define a stagnation state at the initial static state.

$$q - w = \Delta h + \Delta ke + \Delta pe$$

$$q = 0;\ \ w = 0;\ \ ke_o = 0;\ \ pe_o = 0;\ \ \Delta h = h_o - h_i$$

$$h_o = (h_i + ke_i + pe_i) --- 1$$

In the above eq. subscript *o* represents stagnation state and subscript *i* represents initial static state

If subscript *i* is removed from the initial static state

$$h_o = (h + ke + pe) --- 2$$

Thus stagnation state has been expressed as the sum of three static properties. Since the process is isentropic, the final entropy is same as initial enthalpy. Final entropy is stagnation and initial entropy is static.

$$s_o = s --- 3$$

Any two independent properties at a specific state is sufficient to fix the state of simple compressible substance by using thermodynamic relations.

a) Incompressible Fluid: (Density is constant)

$$T\,ds = dh - vdp$$

$$but\ \ ds = 0\ \ because\ \ s_o = s$$

$$dh = vdp$$

$$\int^{o} dh = \frac{1}{\rho}\int^{o} dp$$

Final state is stagnation with subscript *o* and initial state is Static without subscript

$$h_o - h = \frac{1}{\rho}(p_o - p)$$

$$p_o = \rho(h_o - h) + p$$

$$p_o = \rho(h + ke + pe - h) + p$$

$$p_o = \rho(\frac{V^2}{2} + gZ) + p$$

Thus stagnation pressure of an incompressible fluid is expressed in terms of static pressure, velocity and height above a datum line.

$$T \cdot ds = du + pdv$$

$$ds = 0 \quad as \ s_o = s$$

$$du = -pdv$$

$$dv = 0 \quad \rho = constant$$

$$\therefore \quad du = 0; \ u_o = u$$

$$Also, \quad du = C_v dT$$

$$\therefore \quad dT = 0; \ T_o = T$$

Thus using thermodynamic relations stagnation pressure, Stagnation pressure, stagnation temperature and stagnation internal energy are found.

Problem 1

Liquid water at standard density flows at a temperature of 20˚, a static pressure of 10 bar and a velocity of 20m/s. Find the total pressure and total temperature of the water.
Given: T=20˚, p=10 bar and V=20m/s, To=? and Po=? , Water is incompressible with ρ = 1000 kg/m3.

$$p_o = \rho(\frac{V^2}{2} + gZ) + p$$

$$p_o = 1000\left(\frac{20^2}{2}\right) + 10 \times 10^5$$

$$p_o = 12bar$$

$$T_o = T = 20^\circ$$

Problem 2

A turbomachine handling liquid water is located 8m above the sump level and delivers the liquid to a tank located 15m above the pump. The water velocities in the inlet and outlet pipes are 2m/s and 4m/s respectively. Find the power required to drive the pump if it delivers 100 kg/min of water.

Given: $z1$=8m; $z2$=15m; $V1$=2m/s; $V2$=4m/s; mass flow rate=100 kg/min

Find the Power P=?

$$w = q - \Delta h_o = -\frac{\Delta p_o}{\rho}$$

$$w = -\left[\left(\frac{p_2 - p_1}{\rho}\right) + \left(\frac{V_2^{\,2} - V_1^{\,2}}{2}\right) + g(z_2 - z_1)\right]$$

$$w = -\left[0 + \left(\frac{4^2 - 2^2}{2}\right) + 9.81(15 + 8)\right] = -231.6 \ J/kg$$

$$work\ on\ pump = 231.6 \ J/kg$$

$$Power = P = \dot{m}w = 386 \ W$$

b) Perfect gas

$$\frac{c_p}{c_v} = \gamma \ \ and \ \ c_p - c_v = R$$

$$eliminating\ c_v$$

$$c_p = \frac{R\gamma}{\gamma - 1}$$

$$T_o = T + \frac{V^2}{2c_p}$$

Substituting the value of cp in the above equation and simplifying

$$T_o = T\left[1 + \frac{(\gamma - 1)M^2}{2}\right]$$

$$\frac{p_o v_o}{T_o} = \frac{pv}{T}$$

$$\frac{T_o}{T} = \frac{p_o v_o}{pv}$$

$$p_o v_o = pv\left[1 + \frac{(\gamma - 1)M^2}{2}\right]$$

$$\frac{v}{v_o} = \left(\frac{p_o}{p}\right)^{\frac{1}{\gamma}}$$

$$\frac{p_o}{p} = \left(1 + \frac{(\gamma - 1)M^2}{2}\right)^{\frac{\gamma}{\gamma - 1}}$$

Efficiencies of Turbomachines

Efficiency of a turbomachine is given by

$$\eta = \eta_a \eta_m$$

where, η_a = adiabatic efficiency; η_m = mechanical efficiency

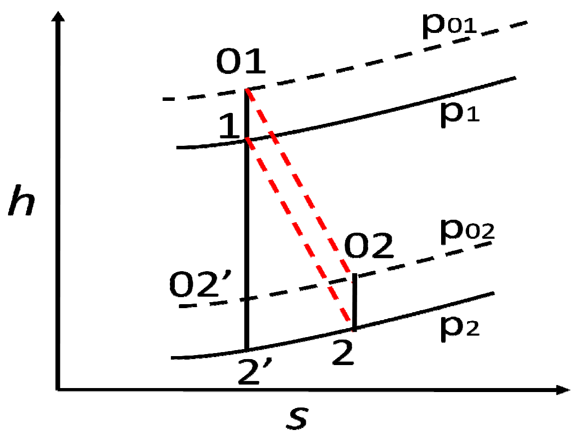
η_m are almost 100%

$$\therefore \ \eta \approx \eta_a$$

The adiabatic efficiency of a TM can be calculated from the h-s diagram for both the expansion and compression process. The ideal work input or output can be using either static or stagnation states.

a) Power Generating Turbomachines (PGTM)

Actual work output for PGTM = $h_{01} - h_{02}$



$$w_{t-t} = h_{01} - h_{02'} \qquad \eta_{t-t} = \frac{h_{01} - h_{02}}{h_{01} - h_{02'}}$$

$$w_{t-s} = h_{01} - h_{2'} \qquad \eta_{t-s} = \frac{h_{01} - h_{02}}{h_{01} - h_{2'}}$$

$$w_{s-t} = h_{1} - h_{02'} \qquad \eta_{s-t} = \frac{h_{01} - h_{02}}{h_{1} - h_{02'}}$$

$$w_{s-s} = h_{1} - h_{2'} \qquad \eta_{s-s} = \frac{h_{01} - h_{02}}{h_{1} - h_{2'}}$$

The proper equation is determined by the conditions of Turbomachine in question.

For example in a turbine if the inlet *ke* is negligible and exit *ke* is used for production of mechanical energy somewhere else, then static to total definition is used. If the exit *ke* is wasted then static to static definition is used.

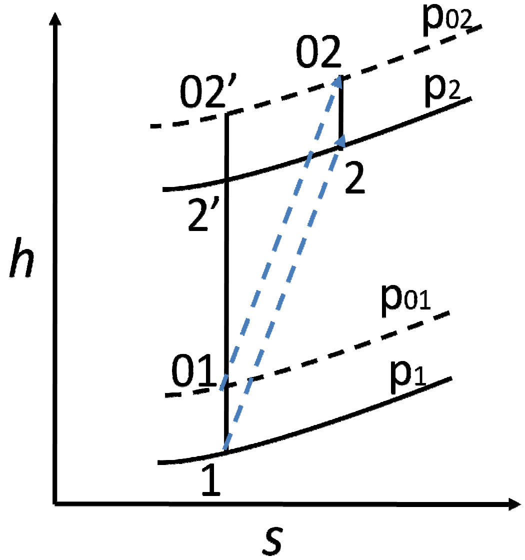
$$\text{Let } h_{01} = 50; \; h_1 = 48; \; h_{02} = 20; \; h_{02'} = 15; \; h_2 = 10; \; h_{2'} = 5$$

$$\eta_{t-t} = \frac{h_{01} - h_{02}}{h_{01} - h_{02'}} = \frac{30}{35} = 85.71\% \qquad \eta_{t-s} = \frac{h_{01} - h_{02}}{h_{01} - h_{2'}} = \frac{30}{45} = 66.67\%$$

$$\eta_{s-t} = \frac{h_{01} - h_{02}}{h_{1} - h_{02'}} = \frac{30}{33} = 90.9\% \qquad \eta_{s-s} = \frac{h_{01} - h_{02}}{h_{1} - h_{2'}} = \frac{30}{43} = 69.76\%$$

b) Power Absorbing Turbomachines (PATM)

Actual work input for PATM = $h_{02} - h_{01}$



$$w_{t-t} = h_{02'} - h_{01} \quad \eta_{t-t} = \frac{h_{02'} - h_{01}}{h_{02} - h_{01}}$$

$$w_{t-s} = h_{2'} - h_{01} \quad \eta_{t-s} = \frac{h_{2'} - h_{01}}{h_{02} - h_{01}}$$

$$w_{s-t} = h_{02'} - h_{1} \quad \eta_{s-t} = \frac{h_{02'} - h_{1}}{h_{02} - h_{01}}$$

$$w_{s-s} = h_{2'} - h_{1} \quad \eta_{s\text{-}s} = \frac{h_{2'} - h_{1}}{h_{02} - h_{01}}$$

Problem 1

Air as a perfect gas flows in a duct at a velocity of 60 m/s, a static pressure of 2 atm., and a static temperature of 300 K. (a) Find total pressure and total temperature of air at this point in the duct. Assume ratio of specific heats as 1.4. (b) Repeat the problem with a flow velocity of 500 m/s.

Given: (a) V=60 m/s, p=2atm., T=300K (b) V=500 m/s, p=2atm., T=300K

Find (a) To = ? and po=? (b) To = ? and po=?

Solution: Equations used are

$$T_o = T + \frac{V^2}{2c_p} \qquad \frac{p_o}{p} = \left(1 + \frac{(\gamma - 1)M^2}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_0 = 300 + \frac{60^2}{2 \times 1005} = 301.79K$$

$$M = \frac{V}{\sqrt{\gamma RT}} = \frac{60}{\sqrt{1.4 \times 287 \times 300}} = 0.1728$$

$$p_0 = 2.07 \ \ bar$$

$$T_0 = 300 + \frac{500^2}{2 \times 1005} = 424.38K$$

$$M = \frac{V}{\sqrt{\gamma RT}} = \frac{500}{\sqrt{1.4 \times 287 \times 300}} = 1.44$$

$$p_0 = 6.9 \ bar$$

Problem 2

Air enters a compressor at a static pressure of 15 bar, static temperature of 15˚C and flow velocity of 50 m/s. At exit, the static pressure is 30 bar, static temperature of 100˚C and flow velocity of 100 m/s. The outlet is 1 m above the inlet. Find a) isentropic change in total enthalpy and b) Actual change in total enthalpy.

Given: p1=15 bar; T1=288K; V=50 m/s; p2=30 bar; T2=100˚C; V2=100m/s; z2-z1=1m.

Find a) Isentropic change in total enthalpy, b) Actual change in total enthalpy

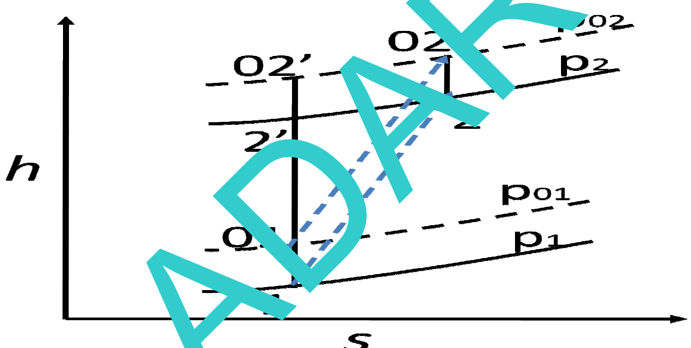
Solution: Plot the h-s or T-s diagram as shown in fig.

p1=15 bar; T1=288K; V=50 m/s; p2=30 bar; T2=100˚C; V2=100m/s; z2-z1=1m

$$Isentropic\ change\ in\ total\ enthalpy = h_{02'} - h_{01}$$

$$h_{02'} - h_{01} = c_p(T_{02'} - T_{01})$$

$$= c_p\left[\left(T_{2'} + \frac{V_2^2}{2c_p} + \frac{gz_2}{c_p}\right) - \left(T_1 + \frac{V_1^2}{2c_p} + \frac{gz_1}{c_p}\right)\right]$$



$$\frac{T_{2'}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} ; \quad T_{2'} = 351.07K$$

$$h_{02'} - h_{01} = c_p(T_{02'} - T_{01}) = 67084 \ KJ/kg$$

$$\textit{Actual change in total enthalpy} = h_{02} - h_{01}$$

$$h_{02} - h_{01} = c_p(T_{02} - T_{01})$$

$$= c_p\left[\left(T_2 + \frac{V_2^{\,2}}{2c_p} + \frac{gz_2}{c_p}\right) - \left(T_1 + \frac{V_1^{\,2}}{2c_p} + \frac{gz_1}{c_p}\right)\right]$$

$$h_{02} - h_{01} = c_p(T_{02} - T_{01}) = 89099.8 \ KJ/kg$$

Finite stage efficiency

1. A stage with a finite pressure drop is a finite stage
2. In a multi-stage turbine along with the overall isentropic efficiency the efficiencies of individual stages are important
3. On account of large pressure drop and associated thermodynamic effect the overall isentropic efficiency is not a true index of aerodynamic or hydraulic performance of machine
4. Different stages with the same pressure drop located in different regions of h-s plane will give different values of work output

Effect of Reheat (Turbines)

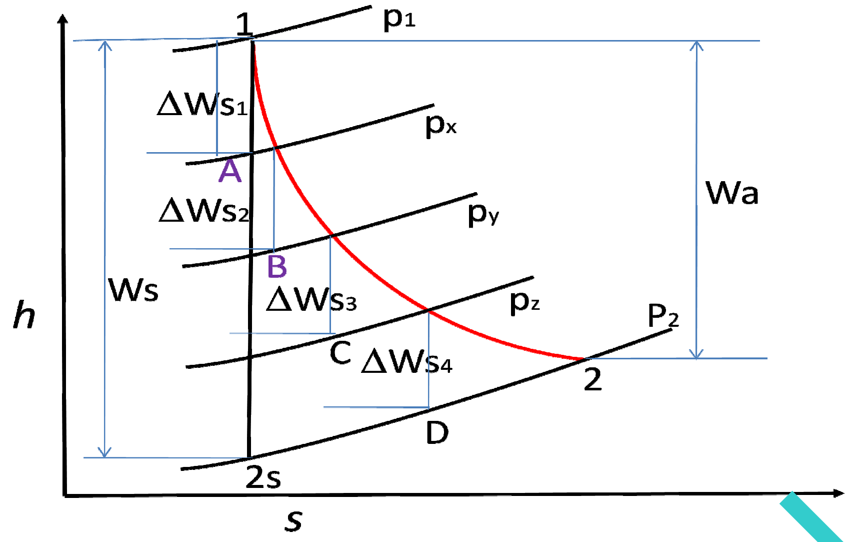
Consider four number of stages between two states as shown in fig. It is assumed that the pressure ratio and stage efficiency are same for all the four stages.

$$\frac{p_1}{p_x} = \frac{p_x}{p_y} = \frac{p_y}{p_z} = \frac{p_z}{p_2} = Constant \text{ - - - } 1$$

$$\textit{Overall efficiency} = \eta_T = \frac{W_a}{W_s}$$

The actual work during expansion from state 1 to state 2 is

$$W_a = \eta_T W_s \text{ - - - } 2$$



The values of ideal or isentropic work in the stages are

$$\Delta W_{s1}, \Delta W_{s2}, \Delta W_{s3}, \Delta W_{s4}$$

The total value of actual work done in these stages are

$$W_a = \sum \Delta W_a = \eta_{st}(\Delta W_{s1} + \Delta W_{s2} + \Delta W_{s3} + \Delta W_{s4}) --- 3$$

$$W_a = \eta_T W_s --- 2$$

Equating eq. 2 and eq. 3

$$W_a = \eta_T W_s = \eta_{st} \sum_{i=1}^{4} \Delta W_{si}$$

$$\eta_T = \eta_{st} \frac{\sum_{i=1}^{4} \Delta W_{si}}{W_s} --- 4$$

The slope of constant pressure lines on h-s plane is given by

$$\left(\frac{\partial h}{\partial s}\right)_p = T --- 5$$

The above equation shows that constant pressure lines must diverge towards the right

$$\therefore \frac{\sum_{i=1}^{4} \Delta W_{si}}{W_s} > 1 \text{---} 6$$

This makes the overall efficiency of the turbine greater than the individual stage efficiencies

$$\eta_T > \eta_{st}$$

The quantity $\frac{\sum_{i=1}^{4} \Delta W_{si}}{W_s}$ is known as reheat factor and is always greater than one.

Reheat is due to the reappearance of stage losses as increased enthalpy during constant pressure heating process.

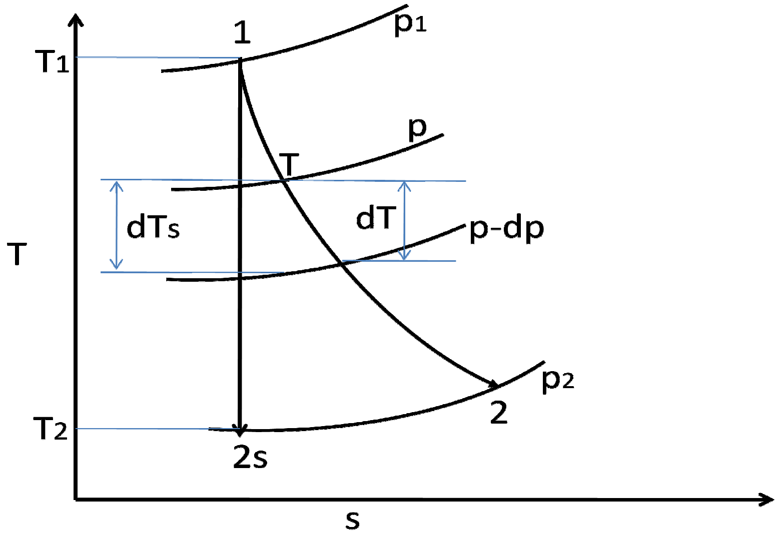
$$RF = \frac{\eta}{\eta_{st}} > 1$$

Infinitesimal stage efficiency or Polytropic efficiency (Turbines)

- To obtain the true aerodynamic performance of a stage the concept of small or infinitesimal stage is used
- This is an imaginary stage with infinitesimal pressure drop and is therefore independent of reheat effect
- Fig. shows a small stage between pressures *p* and *p-dp*

The efficiency of this stage is

$$\eta_p = \frac{\text{actual temperature drop}}{\text{isentropic temperature drop}} = \frac{dT}{dT_s}$$



For infinitesimal isentropic expansion

$$\frac{T-dT_s}{T}=\left(\frac{p-dp}{p}\right)^{\frac{\gamma-1}{\gamma}}$$

$$1-\frac{dT_s}{T}=\left(1-\frac{dp}{p}\right)^{\frac{\gamma-1}{\gamma}}$$

Expanding the terms on r.h.s. using binomial expression and neglecting terms beyond second

$$1-\frac{dT_s}{T}=1-\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{dp}{p}\right)$$

$$\frac{dT_s}{T}=\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{dp}{p}\right)$$

$$But\ \eta_p=\frac{dT}{dT_s}---1$$

$$\therefore \eta_p=\frac{dT}{T}\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p}{dp}\right)$$

$$\frac{dT}{T}=\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{dp}{p}\right)\eta_p---2$$

Integrating the eq.2 between limits 1 and 2

$$\int_{T_1}^{T_2}\frac{dT}{T}=\eta_p\left(\frac{\gamma-1}{\gamma}\right)\int_{p_1}^{p_2}\frac{dp}{p}$$

$$\log_e\frac{T_2}{T_1}=\left(\frac{\gamma-1}{\gamma}\right)\eta_p\log_e\frac{p_2}{p_1}$$

$$\eta_p=\frac{\left(\frac{\gamma}{\gamma-1}\right)\log_e\left(\frac{T_2}{T_1}\right)}{\log_e\left(\frac{p_2}{p_1}\right)}---3$$

$$\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)\eta_p}---4$$

The irreversible adiabatic (actual) expansion process can be considered as equivalent to a polytropic process with index n.

$$\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)\eta_p}=\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)}$$

Equating indices

$$\left(\frac{\gamma-1}{\gamma}\right)\eta_p=\frac{n-1}{n}$$

$$\eta_p=\left(\frac{n-1}{n}\right)\left(\frac{\gamma}{\gamma-1}\right)---5$$

The index of expansion in actual process is

$$n=\frac{\gamma}{\gamma-(\gamma-1)\eta_p}---6$$

When ηp=1, $n=\gamma$ the expansion line coincides with the isentropic expansion. The efficiency of a finite stage can now be expressed in terms of small stage efficiency. Taking static values of T and p and assuming perfect gas

$$\eta_{st} = \frac{T_1 - T_2}{T_1 - T_{2s}}$$

$$T_1 - T_2 = T_1\left(1 - \frac{T_2}{T_1}\right) = T_1\left(1 - \frac{1}{(Pr)^{\frac{\gamma}{\gamma-1}\eta_p}}\right)$$

$$T_1 - T_{2s} = T_1\left(1 - \frac{T_{2s}}{T_1}\right) = T_1\left(1 - \frac{1}{(pr)^{\frac{\gamma-1}{\gamma}}}\right)$$

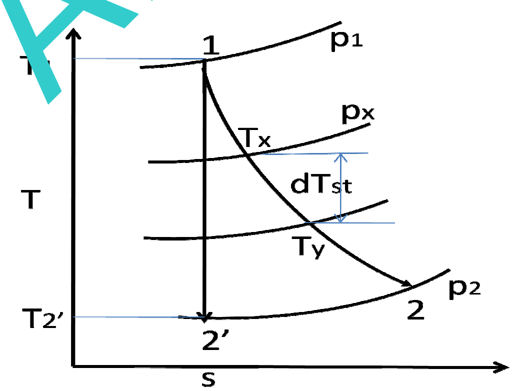
$$\eta_{st} = \frac{1 - pr^{\left(-\frac{\gamma-1}{\gamma}\right)\eta_p}}{1 - pr^{\left(-\frac{\gamma-1}{\gamma}\right)}} ;\text{---}7 \quad \textit{where} \quad pr = \frac{p_1}{p_2}$$

$$\textit{if pressure ratio } pr = \frac{p_2}{p_1}; \quad \eta_{st} = \frac{1 - pr^{\frac{\gamma-1}{\gamma}\eta_p}}{1 - pr^{\frac{\gamma-1}{\gamma}}} \text{--}8$$

Equations 7 and 8 can give the efficiencies of various finite expansion processes with different values of pressure ratio and small stage efficiency

Problem 1

The overall pressure ratio through a 3 stage gas turbine is 10 and efficiency is 86%. The temperature at inlet is 1400 K. If the temperature rise in each stage is same, determine for each stage (a) pressure ratio (b) stage efficiency.
Given: T1=1400 K; p1/p2=10; η=86%; ΔTst= constant
To find: pressure ratio for each stage and ηst
Draw the T-s diagram and assume the gas to be perfect gas



$$\eta = \frac{T_1 - T_2}{T_1 - T_{2'}} = 0.86$$

$$\frac{T_{2'}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{10}\right)^{\frac{0.4}{1.4}}; \ T_{2'} = 725.1K$$

$$\therefore T_2 = 819.6K$$

$$T_1 - T_2 = 580.37K$$

Temperature rise in each stage is same

$$\Delta T_{st} = \frac{580.37}{3} = 193.45K$$

$$T_x = 1206.55K \text{ and } T_y = 1013.1K$$

$$\eta_p = \frac{\left(\frac{\gamma}{\gamma-1}\right)\log_e\left(\frac{T_2}{T_1}\right)}{\log_e\left(\frac{p_2}{p_1}\right)} = \frac{1.4}{0.4} \times \frac{\log_e \frac{819.6}{1400}}{\log_e\left(\frac{1}{10}\right)} = 0.814$$

$$\frac{T_x}{T_1} = \left(\frac{p_x}{p_1}\right)^{\frac{\gamma-1}{\gamma}\eta_p}$$

$$\frac{p_x}{p_1} = \left(\frac{T_x}{T_1}\right)^{\frac{\gamma}{(\gamma-1)\eta_p}} = \left(\frac{1206.55}{1400}\right)^{\frac{1.4}{0.814\times0.4}} = 0.527$$

$$\frac{p_1}{p_x} = 1.895$$

First stage efficiency is given by

$$\eta_{st} = \frac{1 - pr^{\left(-\frac{\gamma-1}{\gamma}\right)\eta_p}}{1 - pr^{\left(-\frac{\gamma-1}{\gamma}\right)}}; \quad \text{where } \ pr = \frac{p_1}{p_x}$$

$$\eta_{st} = 0.8$$

Pressure ratio and stage efficiency of 2 stage is given by

$$T_x = 1206.55K; \; T_y = 1013.1K$$

$$\frac{p_y}{p_x} = \left(\frac{T_y}{T_x}\right)^{\frac{\gamma}{(\gamma-1)\eta_p}} = \left(\frac{1013.1}{1206.55}\right)^{\frac{1.4}{0.814\times.4}} = 0.471$$

$$\frac{p_y}{p_x} = 0.471; \; \frac{p_x}{p_y} = 2.12 \qquad \eta_{st} = \frac{1 - pr^{\left(-\frac{\gamma-1}{\gamma}\right)\eta_p}}{1 - pr^{\left(-\frac{\gamma-1}{\gamma}\right)}}; \;\; where \;\; pr = \frac{p_x}{p_y}$$

$$\eta_{st} = 0.797$$

Pressure ratio and stage efficiency of 3 stage is given by

$$T_y = 1013.1K; \; T_2 = 819.6K$$

$$\frac{p_2}{p_y} = 0.402 \;\; \frac{p_y}{p_2} = 2.487$$

$$\eta_{st} = 0.7938$$

Problem 2

In a 3 stage turbine the pressure ratio of each stage is 2 and stage efficiency is 75%. Calculate the overall efficiency and power developed if the air is initially at a temperature of 600˚C and flows through it at the rate of 25kg/s. Also Find the reheat factor.

Given: ηst=75%; 3 stage turbine; pressure ratio across each stage=2; T1=600˚C; mass flow rate = 25 kg/s To find: η=?; P=?; RF=?

Draw the T-s diagram

$$\eta_{st} = \frac{1 - pr^{\frac{\gamma-1}{\gamma}\eta_p}}{1 - pr^{\frac{\gamma-1}{\gamma}}} = 0.75$$

$$\frac{1 - (0.5)^{.2857 \times \eta_p}}{1 - (0.5)^{0.2857}} = 0.75$$

$$\eta_p = 73.07\%$$

$$\eta = \frac{T_1 - T_2}{T_1 - T_{2'}} = \frac{1 - \left(\frac{p_x}{p_1} \times \frac{p_y}{p_x} \times \frac{p_2}{p_y}\right)^{\frac{\gamma-1}{\gamma}\eta_p}}{1 - \left(\frac{p_x}{p_1} \times \frac{p_y}{p_x} \times \frac{p_2}{p_y}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1 - \left(\frac{p_x}{p_1}\right)^{\frac{\gamma-1}{\gamma}\eta_p \times 3}}{1 - \left(\frac{p_x}{p_1}\right)^{\frac{\gamma-1}{\gamma} \times 3}}$$

$$\eta = 78.62\%$$

$$P = \dot{m}c_p(T_1 - T_2) = 25 \times 1.005 \times 873 \times \left(1 - \frac{1}{8}\right)^{.7307 \times .2857} = 7724.25KW$$

$$RF = \frac{\eta}{\eta_{st}} = \frac{0.7862}{0.75} = 1.048$$

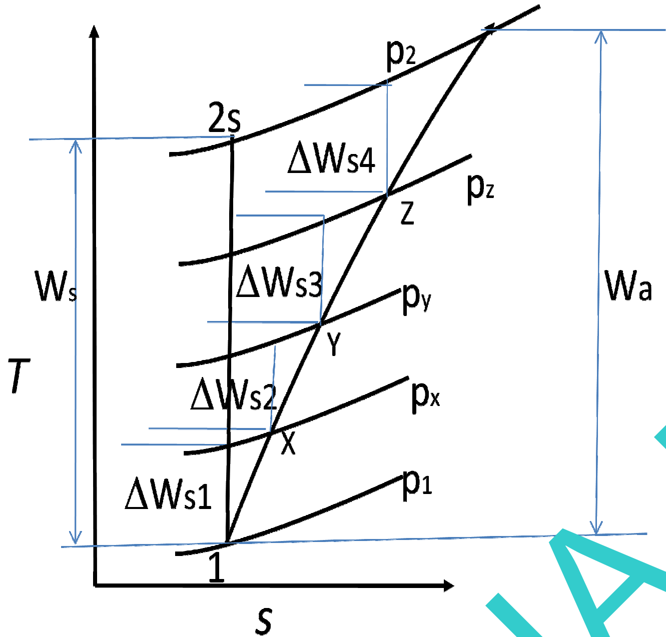
Finite stage efficiency (Compressor)

- A compressor with a finite pressure rise is known as a finite stage
- Stage work is a function of initial temperature and pressure ratio
- For the same pressure ratio, a stage requires a higher value of work with higher temperature
- Thus compressor stages in the higher temperature region suffer on account of this

The above factors have a cumulative effect on the efficiency of multistage compressor

Effect of preheat (Compressor)

Consider a compressor with four stages as shown. It is assumed that all the stages have the same efficiencies and pressure ratios.



The total isentropic work from state 1 to 2s is Ws.
The isentropic work in the individual stages are ΔWs1, ΔWs2, ΔWs3 and ΔWs4
The overall efficiency of the compressor is η=Ws/Wa

$$\textbf{However } \eta_{st} = \frac{\Delta W_{s1}}{W_{1x}} = \frac{\Delta W_{s2}}{W_{xy}} = \frac{\Delta W_{s3}}{W_{yz}} = \frac{\Delta W_{s4}}{W_{z2}};$$

$$W_a = \frac{1}{\eta_{st}} \sum_{i=1}^{4} \Delta W_{si} \qquad W_a = \frac{1}{\eta_{st}} = (\Delta W_{s1} + \Delta W_{s2} + \Delta W_{s3} + \Delta W_{s4})$$

$$\textbf{but } \frac{W_s}{\sum_{i=1}^{4} \Delta W_{si}} < 1$$

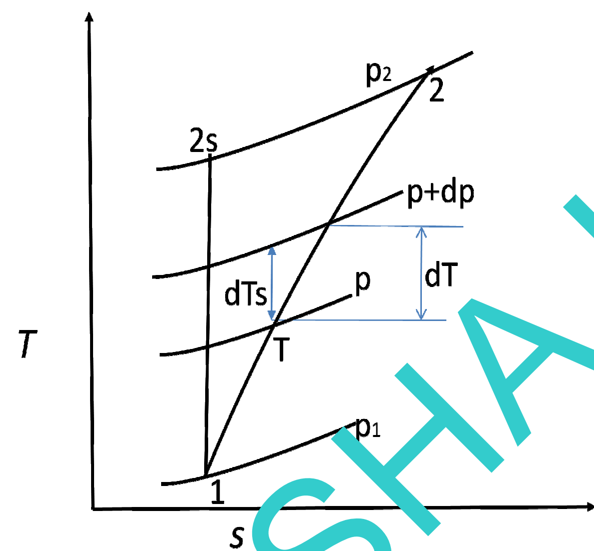
This makes the overall efficiency of the compressor smaller than stage efficiency

$$\eta < \eta_{st}$$

This is due to the thermodynamic effect called ‘Pre-reheating ; the gas is not intentionally heated(preheated) at the end of each compression stage. The preheat in small constant pressure processes is only an internal phenomena and the compression process still remains an adiabatic process.

Infinitesimal or Polytropic Efficiency (Compressor)

- A finite compressor stage can be made up of infinite number of small stages
- Each of these infinitesimal stages have an efficiency ηp called polytropic efficiency or infinitesimal stage efficiency
- It is independent of thermodynamic effect and is therefore a true measure of the aerodynamic performance of the compressor
- Consider a stage in which air is compressed from state 1 to state 2. It also shows an infinite stage operating between pressures p and $p+dp$



$$\eta_p = \frac{dT_s}{dT}$$

$$\frac{T + dT_s}{T} = \left(\frac{p + dp}{p}\right)^{\frac{\gamma-1}{\gamma}}$$

$$1 + \frac{dT_s}{T} = \left(1 + \frac{dp}{p}\right)^{\frac{\gamma-1}{\gamma}}$$

using bionomial expansion

$$1 + \frac{dT_s}{T} = 1 + \left(\frac{\gamma - 1}{\gamma}\right)\frac{dp}{p}$$

$$\frac{dT_s}{T} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{dp}{p}$$

Substituting the value of *dTs* into equation 1

$$\eta_p \frac{dT}{T} = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{dp}{p}\right)$$

$$\frac{dT}{T} = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{dp}{p}\right) \times \frac{1}{\eta_p} --2$$

Integrating eq.2 between state 1 to 2

$$\log_e \frac{T_2}{T_1} = \left(\frac{\gamma-1}{\gamma}\right)\frac{1}{\eta_p}\log_e \frac{p_2}{p_1} \quad or \quad \eta_p = \frac{\left(\frac{\gamma-1}{\gamma}\right)\log_e \frac{p_2}{p_1}}{\log_e \frac{T_2}{T_1}} --3$$

Assuming the irreversible adiabatic compression is equivalent to a polytropic process with index n, equation 3 can be written as

$$\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma\eta_p}} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}$$

$$\frac{\gamma-1}{\gamma\eta_p} = \frac{n-1}{n}$$

$$\eta_p = \frac{\gamma-1}{\gamma} \times \frac{n}{n-1} = \frac{\gamma\eta_p}{1-\gamma(1-\eta_p)} ---4$$

The efficiency of finite compressor stage can be related to small stage efficiency
The actual temperature rise is given by

$$T_2 - T_1 = T_1\left(\frac{T_2}{T_1} - 1\right) = T_1\left(pr^{\frac{\gamma-1}{\gamma\eta_p}} - 1\right) \quad where \ pr = \frac{p_2}{p_1}$$

$$\eta_{st} = \frac{T_{2s}-T_1}{T_2-T_1} = \frac{\left(\frac{T_{2s}}{T_1}-1\right)}{\left(\frac{T_2}{T_1}-1\right)} = \frac{pr^{\frac{\gamma-1}{\gamma}}-1}{pr^{\frac{\gamma-1}{\gamma\eta_p}}-1} ---5$$

Problem1

A 16 stage axial flow compressor is to have a pressure ratio of 6.3, with a stage efficiency of 89.5%. Intake conditions are 288K and 1 bar. Find (a) Overall efficiency (b) Polytropic efficiency (c) Preheat factor. Assume pressure ratio per stage is same.

Given: 16 stage axial flow compressor, pressure ratio = 6.3, ηst=89.5%, T1=288K and p1=1bar

Solution:

$$\frac{p_{17}}{p_{16}} = \frac{p_{16}}{p_{15}} = \ldots = \frac{p_3}{p_2} = \frac{p_2}{p_1} = constant = x$$

$$\frac{p_{17}}{p_1} = 6.3 = x^{16}$$

$$x = pressure\ ratio\ per\ stage = 1.1219$$

$$\eta_{st} = \frac{(pr)^{\frac{\gamma-1}{\gamma}} - 1}{(pr)^{\frac{\gamma-1}{\gamma\eta_p}} - 1} = 0.895 = \frac{(1.1219)^{0.2857} - 1}{(1.1219)^{0.2857/\eta_p} - 1}$$

$$\eta_p = 0.9045$$

$$\eta = \frac{(pr_o)^{\frac{\gamma-1}{\gamma}} - 1}{(pr_o)^{\frac{\gamma-1}{\gamma\eta_p}} - 1} = \frac{(6.3)^{0.2857} - 1}{(6.3)^{0.2857/\eta_p} - 1} = 87.75\%$$

$$Preheat\ factor = \frac{\eta}{\eta_{st}} = \frac{0.8775}{0.9095} = 0.97$$

Problem 2

An air compressor has 8 stages of equal pressure ratios of 1.3. The flow rate through the compressor and its overall efficiency are 45 kg/s and 80% respectively. If the conditions of air at entry are 1 bar and 35˚C determine (a) State of compressed air at exit (b) polytropic efficiency (c) Stage efficiency

Given: 8 stages of equal pressure ratio of 1.3, mass flow rate of 45 kg/s, η = 80%, $p1$= 1bar, $T1$= 35˚C

To find: (a) $p2$=?, $T2$=? (b) ηp=? (c) ηst=?

Stage pressure ratio = 1.3;
Overall pressure ratio = (1.3) = 8.157

$$\frac{T_{2'}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{8.157}{1}\right)^{0.2857}$$

$$T_{2'} = 561K$$

$$\eta = \frac{T_{2'} - T_1}{T_2 - T_1} = 0.8 = \frac{561 - 308}{T_2 - 308}$$

$$T_2 = 624.26K \quad p_2 = 8.157bar$$

$$\eta_p = \frac{\left(\frac{\gamma-1}{\gamma}\right)\log_e \frac{p_2}{p_1}}{\log_e \frac{T_2}{T_1}} = 0.2857\frac{\log_e\left(\frac{8.157}{1}\right)}{\log_e\left(\frac{624.26}{308}\right)} = 84.88\%$$

$$\eta_{st} = \frac{(pr)^{\frac{\gamma-1}{\gamma}} - 1}{(pr)^{\frac{\gamma-1}{\gamma\eta_p}} - 1} = \frac{1.3^{0.2857} - 1}{1.3^{\frac{0.2857}{0.8488}} - 1} = 84.3\%$$